First Order Logic Reading Guide

For those not trained in mathematics, formal logical expressions are hard to decipher and comprehend. In this text, we have chosen a particularly compact symbolic form, in order to visualize more clearly the essential inferences that the expressions describe.

However, all logical expressions can be brought into a sufficiently comprehensible linguistic form resolving the logical symbols by stereotype parts of speech when reading them. This works particularly well for short logical expressions.

For more complex logical expressions, an explicit linguistic form may become too extended, and the reader may again loose track of the overall meaning. Logical expressions use variables in different parts to refer to any item for which that part of the expression applies. Sometimes, a natural language rendering may become more comprehensible when relative pronouns (e.g. who, which, whose) are introduced in order to connect such items within an expression in a more compact form. Also, “instance of” can be replaced by “a”/”an”. Using the latter, we show below some more explicit versus more compact reading alternatives.

In the following, we explain the (English) reading method by example of sample definitions from the CIDOC CRM text. Instead of explaining the rules of correspondance between logical symbolism and parts of speech in words, we use blue to denote phrases corresponding to FOL syntactic elements, red and green for variables, black for concept labels, and red for “there exists”. We expand the concept identifiers by the full labels.

*First example: the definition of P11.*

P11 had participant (participated in)

Domain:

[E5](#_toc7383) Event

Range:

[E39](#_toc8005) Actor

Subproperty of:

[E5](#_toc7383) Event. [P12](#_toc9032) occurred in the presence of (was present at): [E77](#_toc8517) Persistent Item

**In First Order Logic:**

**P11(x,y) ⇒ E5(x)**

**P11(x,y) ⇒ E39(y)**

**P11(x,y) ⇒ P12(x,y)**

*Read:*

**P11(x,y) ⇒**

 **E5(x)**

*as:*

**If a particular x is related to another particular y by the property P11 had participant, then**

 **x must be an instance of E5 Event.**

Shorter: **If x** (P11)**had participant y, then x must be an instance of E5 Event.**

This is the FOL form of the domain condition above..

Read:

**P11(x,y) ⇒**

**E39(y)**

*as:*

**If a particular x is related to another particular y by the property P11 had participant, then**

 **y must be an instance of E39 Actor.**

Shorter: **If x** (P11)**had participant y, then y must be an instance of E39 Actor.**

This is the range condition.

**P11(x,y) ⇒**

**P12(x,y)**

**If a particular x is related to another particular y by the property P11 had participant, then**

 **x must also be related to y by the property P12 occurred in the presence of**

Shorter: **If x** (P11)**had participant y, then x (**P12) **occurred in the presence of y.**

this is the “suproperty of” statement.

Note that the FOL statements are ontological, i.e., they refer to the assumed reality as premise, as far as the referred CRM concepts are applicable to this reality, regardless whether we have knowledge of this reality or not.

The “if…then…must be..” constitutes the inference described by and in the direction of the “⇒” arrow (also reading as “implies”).

**Now something more complicated, an existential statement:**

*Second example: the definition of P11.:*

**P8 took place on or within (witnessed)**

Domain:

[E4](#_toc7351) Period

Range:

[E18](#_toc7650) Physical Thing

Quantification:

many to many (0,n:0,n)

Scope note:

This property describes the location of an instance of E4 Period with respect to an instance of E19 Physical Object.

This property is a shortcut of the more fully developed path from E4 Period through *P7 took place at*, E53 Place, *P156i is occupied by* E18 Physical Thing.

It describes a period that can be located with respect to the space defined by an E19 Physical Object such as a ship or a building. The precise geographical location of the object during the period in question may be unknown or unimportant.

For example, the French and German armistice of 22 June 1940 was signed in the same railway carriage as the armistice of 11 November 1918.

Examples:

* The coronation of Queen Elizabeth II (E7) *took place on or within* Westminster Abbey (E18). (Strong, 2005)

**In First Order Logic:**

P8(x,y) ⇒ E4(x)

P8(x,y) ⇒ E18(y)

**P8(x,y) ⇐ (∃z) [E53(z) ˄ P7i(z,x) ˄ P156i(z,y)]**

The last statement above interprets the paragraph highlighted in yellow in the scope note above as a FOL statement.

For reading this properly, you need the property names of P7i, P156i, and their domain and range conditions.

Start reading in the direction of the “⇒” arrow:

**⇐ (∃z) [E53(z) ˄ P7i(z,x) ˄ P156i(z,y)]**

**If there exists a particular z, which is an instance of E53 Place and this z (**P7i) **witnessed x** (which therefore must be instance of E4 Period) **and this z P156i is occupied by y** (which therefore must be instance of E18 Physical Thing)

**…then x** (P8) **took place on or within y**

Note that we use in the parentheses above the domain – range conditions of P7 and P156.

Or more fluent:

**If there exists a particular E53 Place z which** (P7i) **witnessed x (an instance of E4 Period) and** (P156i) **is occupied by y (an instance of E18 Physical Thing),**

**…then this x (P8) took place on or within this y**